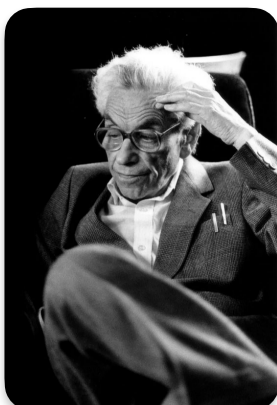


Paul Erdős

(0)



The non-existence of a Hamel-basis and the general solution of Cauchy's functional equation for nonnegative numbers

By J. ACZEL (Debrecen - Wasserloo Ott) and P. ERDŐS (Budapest)

1. It is well known that the general continuous solution of Cauchy's functional equation (1)  $f(x+y) = f(x) + f(y)$  is (see e.g. [1] or [11])  $f(x) = cx$ , with an arbitrary constant  $c$ . As DARBOUX has proved ([4]), this (with nonnegative 'c') is also the most general solution of (1) which is nonnegative for positive 'x' (it is even enough to suppose the nonnegativity for small positive 'x'). But without any regularity-suppositions (2) isn't anymore the most general solution of (1), this can be shown and at the same time the most general solution of (1) can be constructed with the Hamel-basis of real numbers ([7]).

In all these results (1) was supposed valid for all real  $x, y$ , and then also (2) is verified for all real 'x' moreover, the Hamel-basis also gives a representation of the most general continuous solution of (1) also with this restriction and with nonnegative 'c', also the most general solution nonnegative for (small) positive variables. But how to construct in this case the most general solution of (1) for nonnegative 'x, y' Are there Hamel-bases of the nonnegative numbers?

In this little note, we answer the second question in the negative and construct nevertheless the general solution asked for in the first one.

2. We first recall the above-mentioned results of Cauchy, Darboux and Hamel with short proofs. (1) implies by induction (3)  $f(x) + f(x) + \dots + f(x) = f(nx) = nx \cdot c$  and with  $x_1 = x_2 = \dots = x_n = x$  (4)  $f(x) = cf(x)$ .

If now  $x = \frac{m}{n}$  ( $m, n \in \mathbb{N}, n \neq 0$ ), then by (4)  $nf(\frac{m}{n}) = f(m) = mc$ ,  $f(\frac{m}{n}) = \frac{m}{n}c = \frac{m}{n}f(1)$

Aczél, János Dezső, and Paul Erdős (1965): "The Non-Existence of a Hamel-Basis and the General Solution of Cauchy's Functional Equation for Nonnegative Numbers," *Publicationes Mathematicae Debrecen* 12, 259–263.

János Dezső Aczél

(1)



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Results in Mathematics

A FUNCTIONAL EQUATION ARISING FROM SIMULTANEOUS UTILITY REPRESENTATIONS

János Aczél, R. Duncan Luce, and A. A. J. Marley

Abstract

Suppose that two classes of utility representations of preferences over utilities and one increasing in-creases, but nonmonotonic over certain lower-dimensional quantities. The resulting task is the functional equation  $f(x) + g(x) = f(y) + g(y)$  ( $x, y \in \mathbb{R}^n, n \geq 2$ ), and to the recently [12] ( $\mathbb{R}^n \ni x, y \in \mathbb{R}^n$ ), where the function  $f$  is a utility increasing representation of the preference and  $g$  is a utility decreasing representation of the preference over the lower-dimensional quantities.

Mathematics subject classification: 98B01, 98B14  
Key words: Functional equations, Utility representations

Introduction

Consider a preference order  $\succsim$  over the class of finite gambles of the form  $(x, C)$  where  $x \in \mathbb{R}^n$  is a finite set of nonnegative utilities and  $C$  is a convex polytope in  $\mathbb{R}^n$ . We assume that the preference order is represented by a utility function  $f$  on  $\mathbb{R}^n$ . The representation is assumed to be continuous. The utility function  $f$  is assumed to be increasing in each coordinate. The utility function  $f$  is assumed to be increasing in each coordinate. The utility function  $f$  is assumed to be increasing in each coordinate.

Aczél, János Dezső, Robert Duncan Luce, and Anthony Alfred John Marley (2003): "A Functional Equation Arising from Simultaneous Utility Representations," *Results in Mathematics* 43, 193–197.

Anthony Alfred John Marley

(2)



The "Horse Race" Random Utility Model for Choice Probabilities and Reaction Times, and its Competing Risks Interpretation

A. A. J. MARLEY  
AND  
HANS COLONIUS  
University of Bayreuth

Random utility models have traditionally been applied to probabilistic choice data, with little attention to reaction times. We describe the case of "horse race" random utility models that can be applied to choice probabilities and reaction times. We show that any such model can be represented by a single random utility function. We show that any such model can be represented by a single random utility function.

Mathematics subject classification: 98B01, 98B14  
Key words: Functional equations, Utility representations

Marley, Anthony Alfred John, and Hans Colonius (1992): "The 'Horse Race' Random Utility Model for Choice Probabilities and Reaction Times, and its Competing Risks Interpretation," *Journal of Mathematical Psychology* 36 (1), 1–20.

Hans Colonius

(3)



Intersensory facilitation in the motor component?

Adele Diederich and Hans Colonius

Summary: In the binocular facilitation task the observer must respond to a signal in either of two modalities, RT to one or the other. A signal in both modalities leads to a shorter RT than a signal in either modality. This is interpreted as evidence for intersensory facilitation in the motor component. The paper reports on a series of experiments designed to test this hypothesis. The results show that intersensory facilitation is present in the motor component. The results show that intersensory facilitation is present in the motor component.

Mathematics subject classification: 98B01, 98B14  
Key words: Functional equations, Utility representations

Diederich, Adele, and Hans Colonius (1987): "Intersensory Facilitation in the Motor Component?," *Psychological Research* 49 (1), 23–29.

Adele Diederich

(4)



When the Poorest Are Neglected

A Vignette Experiment on Need-Based Distributive Justice

Alexander Max Bauer, Adele Diederich, Stefan Traub, Arne H. Weiss

Abstract: We examine the role of need satisfaction in consumption. In an experiment, we compare three conditions with regard to the utility of goods. In the first condition, we compare the utility of goods with and without need satisfaction. In the second condition, we compare the utility of goods with and without need satisfaction. In the third condition, we compare the utility of goods with and without need satisfaction.

Bauer, Alexander Max, Adele Diederich, Stefan Traub, and Arne Robert Weiss (forthcoming): "Thinking About Need. A Vignette Experiment on Need-Based Distributive Justice," *The Journal of Economic Inequality*.

Alexander Max Bauer

(5)

